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# The prediction of periods of high volatility in exchange markets

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**Abstract.** A statistical connection is identified between the current spread in a market over a given time period and the drift of the market during previous time periods. It is shown that periods of high spread are likely to be preceded by periods with relatively large market drifts. Several markets, including the UK pound per US Dollar, US Dollar per Yen, UK pound per Euro, and the UK FT100 index have been analysed from 1991 to 2000 over variable periods of weeks, months and quarters. Within each period, $i$ the natural logarithm of the daily end-of-trade market value has been least squares fitted to a linear regression line, and evaluations made of the regression line slope $\mu_i$, the direct spread $s_i$ with respect to the mean value, and the regression spread $r_i$ of the deviations from the regression line. Significant correlations have been observed between the current monthly direct spread $s_i$ for each period $i$ and the absolute value of the drifts $|\mu_{i-j}|$ evaluated j periods earlier. This correlation coefficient is as high as 0.746 for a period of one quarter ($j = 1$) and appears to die away after around 9 months for quarterly averages, after around 4 months for monthly averages and after around 2 months for weekly averages.

**PACS.** 89.65.Gh Economics, business, and financial markets

## 1 Introduction

In many dealing situations it is desirable to perform trades at a time when the current market volatility is relatively low. The volatility of an index may be defined as the standard deviation, $\sigma$, of the natural logarithm of the index changes over a given time period. This definition is linked to the random walk model of index movements, with its quasi-Gaussian distribution of the daily market changes. However volatility is usually quoted on an annual basis, so that the standard deviation of the market changes over any period of $N_{\text{days}}$ trading days is $\sigma\sqrt{N_{\text{days}}}/\sqrt{N_{\text{year}}}$ where $N_{\text{year}} = 261$ is the mean number of trading days in a year.

Our paper deals with periods of weeks, months and quarters. Within each period $i$, the daily end-of-trade market values were least-squares fitted to a linear regression line as illustrated in Figure 1, and estimates made of the root-mean-square deviation from the mean value which we shall call the direct spread $s_i$, the deviation from the regression line which we shall call the regression spread $r_i$, and the slope of the regression line $\mu_i$.

Within the random walk model, the mean values of both spreads are related to the volatility. This relationship with volatility was investigated by simulating many long random walks with a given value of $\sigma$ and zero overall drift, evaluating the spreads and slope within each period, and averaging them to fine the mean. For example the mean direct spread appears close to the value $\sigma(N_{\text{days}}/2\pi N_{\text{year}})^{1/2}$. The regression spread appears to be about 2/3 of this value. The magnitude of the slope is correlated with the direct spread. However with simulated random-walks there is no significant correlation either between the magnitude of the slope and the regression spread in any one period, or between either spread and values of the slope in previous periods. Such correlations are shown in real data, and can be used to help estimate current volatility. Figure 1 illustrates the monthly averages for the UK Pounds per Euro rate.

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## 2 Market drift, direct spread and regression spread

Detailed investigations have been made for several exchange markets, including the GBP per USD, GBP per EUR and USD per JPY. Other markets such as the UK, US, EU, Swiss and Japanese interest rates and the Dow Jones, FTSE 100, Nikkei and DAX indices have also been analysed. Indices from 1991 to 2000 have been converted into natural logarithms and normalised on 1/1/00. The process of conversion to natural logarithms means that spreads and drifts from any market index are equal in percentage terms and so are directly comparable. They have then been averaged over variable periods of weeks, months, quarters and years. Figure 2 shows the UK Pounds per US Dollar exchange rate as a function of time over monthly

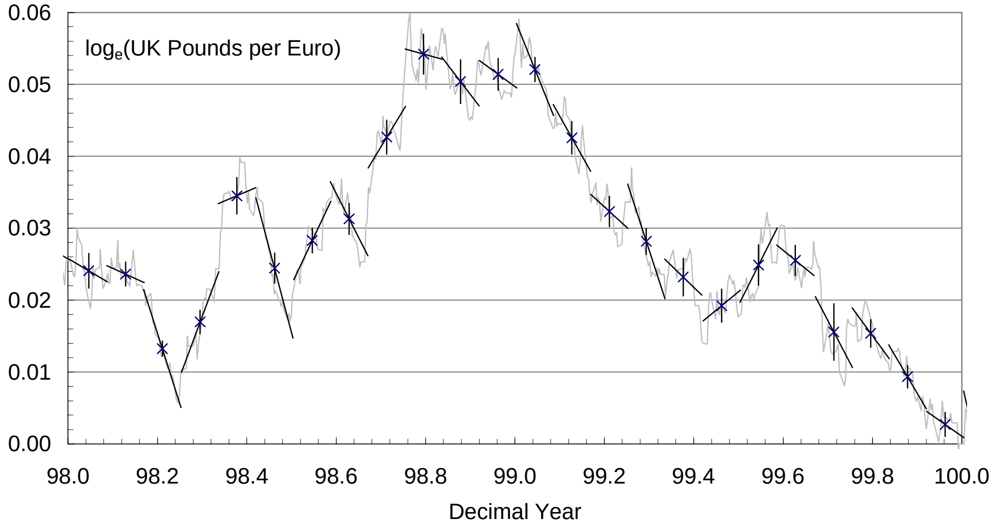


**Fig. 1.** The natural logarithm of the exchange index (feint) giving the UK pounds per Euro, normalised on 1/1/00, and converted from the German Mark at 1 EUR = 1.95583 DEM before 1/1/99. The daily values within each monthly period have been fitted to a linear regression line (full) together with a vertical line denoting the spread with respect to the regression line.

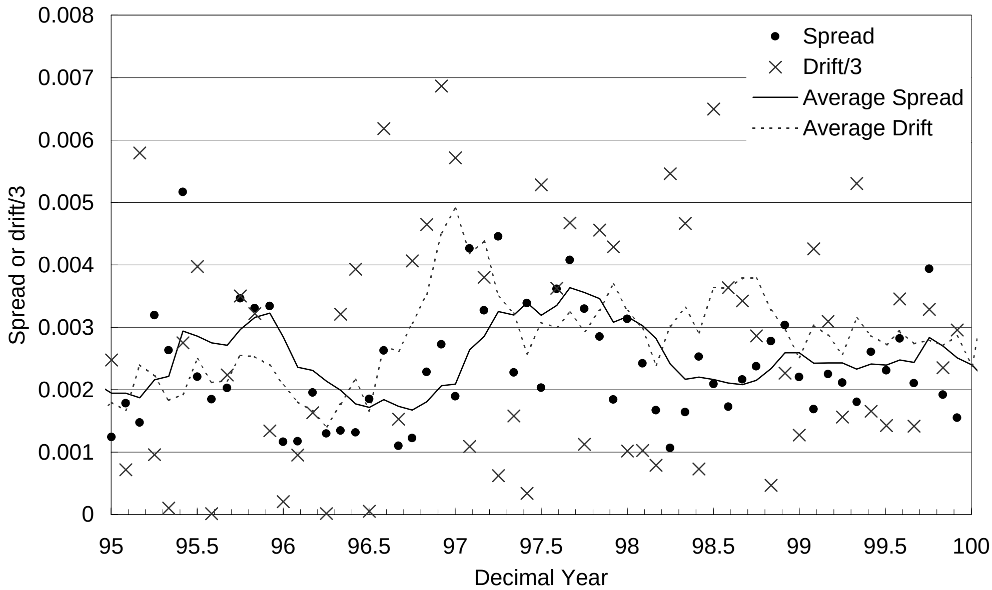


**Fig. 2.** The monthly regression spreads $r_i$ (dark circles) and drifts $|\mu_i|$ (feint crosses) of the exchange index between the UK pound and US dollar. Average values defined later in the text are shown by continuous and dashed lines respectively. Around 1997 a period of generally high spread is preceded by a period of relatively high drift.

periods from 1995 to 2000. There is much scatter in the individual monthly values, but the weighted average values, detailed later and shown by the lines, show significant correlations. Most usefully for prediction purposes, there is some indication that periods of high spread are preceded by periods of high drift. Thus around 1997 in the centre of the figure, the dashed grey line showing the average drift rises rapidly around half a year previous to the rise in the average volatility. The same effect is present in the monthly data points.

## 3 The correlations between historic regression drift and spread

For the purpose of predicting spread it is appropriate to make scatter plots of the direct spreads $s_i$ or regression spreads $r_i$ during a given period $i$ against the absolute value of the drift $|\mu_{i-j}|$, as it was $j$ periods earlier. Figure 3 shows the direct spread $s_i$ against $|\mu_{i-j}|$, for the GBP per USD index, but this time for quarterly periods, with a prior period of $j = 1$ quarter. In fact for any

**Table 1.** The parameters of the fitted linear relationship between the current regression period's direct spread $s_i$ or its regression spread $r_i$ and the drift $\mu_{i-j}$ over the prior monthly or quarterly period $j$ periods before. The correlation coefficient $C_s$ or $C_r$ would be 1 for a perfect positive correlation and 0 for a random distribution. The predictive ratios ($R_s$ or $R_r$) is 1 minus the spread measured from the regression line $r_i$ divided by the overall direct spread $s_{oo}$ or regression spread $r_{oo}$.

| Index | Prior | Direct | | | Regression | | |
|---|---|---|---|---|---|---|---|
| | Period | Slope ($M_s$) | Ratio ($R_s$) | Corr ($C_s$) | Slope ($M_r$) | Ratio ($R_r$) | Corr ($C_r$) |
| GBP per USD | 1 quarter | 4.915 | 0.170 | 0.747 | 3.310 | 0.130 | 0.702 |
| | 2 quarters | 4.549 | 0.147 | 0.723 | 2.668 | 0.090 | 0.643 |
| | 3 quarters | 5.248 | 0.201 | 0.776 | 3.661 | 0.177 | 0.557 |
| | 4 quarters | 3.115 | 0.067 | 0.600 | 1.938 | 0.047 | 0.551 |
| | 5 quarters | 2.232 | 0.033 | 0.505 | 1.788 | 0.039 | 0.526 |
| | 6 quarters | 1.665 | 0.023 | 0.463 | 1.908 | 0.056 | 0.576 |
| | 1 month | 0.972 | 0.087 | 0.639 | 0.390 | 0.068 | 0.601 |
| | 2 months | 0.186 | 0.004 | 0.279 | 0.146 | 0.009 | 0.368 |
| | 3 months | 0.445 | 0.018 | 0.433 | 0.318 | 0.045 | 0.544 |
| | 4 months | 0.331 | 0.010 | 0.377 | 0.345 | 0.053 | 0.566 |
| | 5 months | 0.132 | 0.002 | 0.236 | 0.215 | 0.020 | 0.447 |
| | 6 months | 0.297 | 0.008 | 0.354 | 0.148 | 0.009 | 0.372 |
| USD perYEN | 1 quarter | −1.168 | 0.017 | −0.142 | 0.174 | 0.002 | 0.248 |
| GBP per EUR | 1 quarter | −1.385 | 0.020 | −0.095 | 0.241 | 0.002 | 0.246 |
| USD per YEN | 1 month | 0.463 | 0.019 | 0.439 | 0.674 | 0.079 | 0.625 |
| GBP per EUR | 1 month | 0.405 | 0.061 | 0.585 | 0.284 | 0.007 | 0.347 |

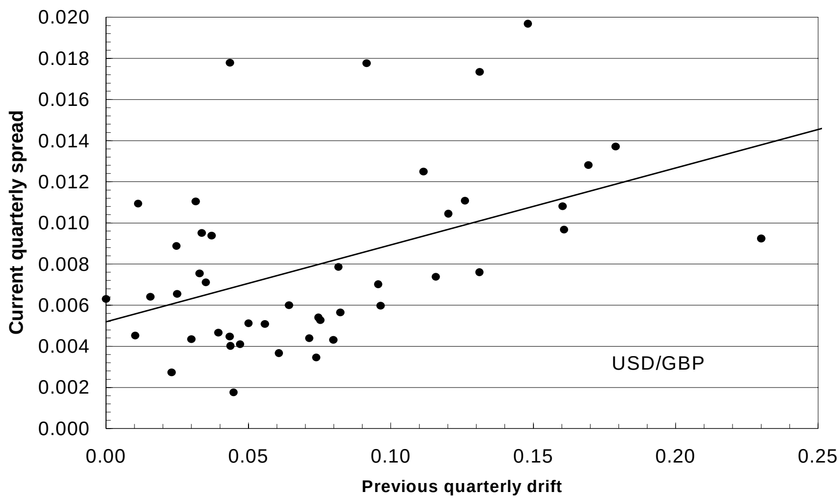


**Fig. 3.** The correlation between quarterly direct spreads $s_i$ for a particular quarter and the drifts $|\mu_{i-j}|$ for the GBP/USD exchange index measured one quarter earlier $j = 1$. The line shows the least squares fit. The parameters of the least squares fitted lines and the correlation coefficients for several times $j$, periods and indices are given in Table 1.

period $j > 0$ the degree of correlation is similar no matter whether the direct spread $s_i$ or the regression spread $r_i$ is used. This is useful in that the direct spread is most directly related to the volatility. The exact results are sensitive to the starting date within the correlation period, but appear to be significant for all starting dates. In most cases there appears to be a significant linear relationship between spread and prior drift so that a regression fit may again be made as shown, and estimates made of the direct slope $M_s$, regression slope $M_r$ and the correlation coefficients $C_s$ or $C_r$. The results for several indices and prior periods are shown in Table 1. It is seen that as the prior period increases, the slope tends to drop, along with the correlation coefficient. The predictive power

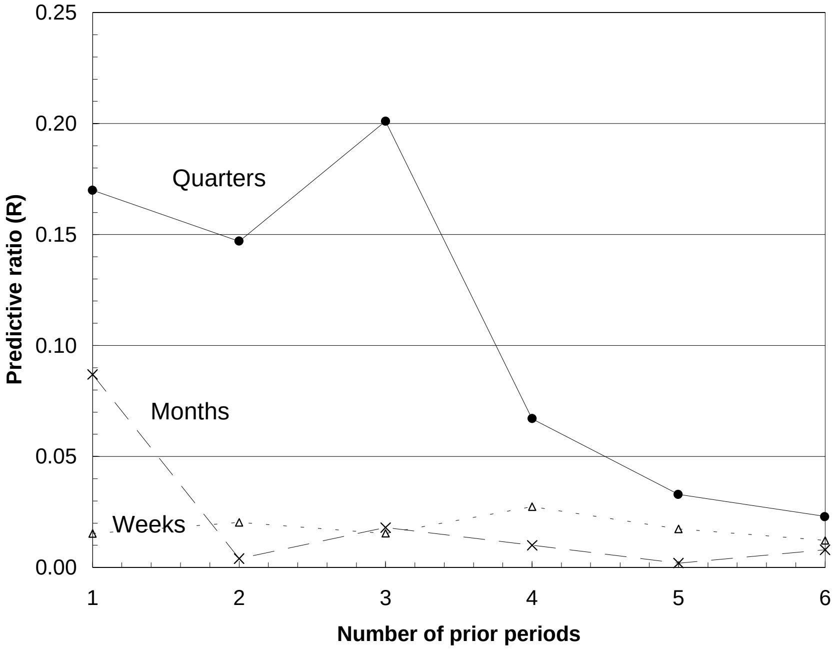


**Fig. 4.** The predictive ratio ($R_s$) given by the spread $r$ from the regression line between the overall spread $s_i$ for a particular period and drifts $|\mu_{i-j}|$ for $j$ periods previous, expressed as a fraction of the overall spread $s_{oo}$, and plotted against the number of prior periods $j$.

of the correlations depends on the difference between the spread from the regression line $r_s$ or $r_r$ and the overall spread $s_{oo}$, or $r_{oo}$. We define predictive ratios $R_s = 1 - s_r/s_{oo}$ and $R_r = 1 - r_r/r_{oo}$. Although there is considerable scatter, the regression spread $r_r$ generally rises with the number of prior periods $j$ towards the overall spread $r_{oo}$, so that the predictive ratios $R_s$ and $R_r$ move towards zero.

## 4 The influence of the prior time period in predicting future spread

Figure 4 shows the predictive ratio ($R_s$) plotted against the number of prior periods for periods of quarters, months and weeks. The predictive ratio appears to drop off significantly as the averaging period decreases. However some of the statistical information can be recovered by combining the drift data from many previous periods. A simple method of allowing for this extended correlation is to define an exponential average drift over a defined number of periods. It suggests that for quarterly periods, a simple average over 3 quarters is appropriate giving a predictive ratio, enhanced over that seen in any one quarter, $R_s = 0.28$. For monthly periods an average over 8 months with a weighting factor of each month over the previous one of 0.7 gives the much lower but still significant predictive ratio $R_s = 0.073$. The statistics for weekly periods are very poor, with all periods having a correlation coefficient below 0.5 and a predictive ratio below 0.03. However an average over 52 weeks with a weighting factor of 0.89 gives the slightly lower predictive ratio $R_s = 0.06$. More advanced statistical approaches, such as principal component analysis could improve these results.

## 5 Conclusions

There is a significant correlation between both the direct spread $s_i$ of the market values measured with respect to the mean measured over a period $i$, and the regression spread $r_i$ with respect to a regression line, to the absolute value of the drifts $|\mu_{i-j}|$ measured $j$ prior periods previously. In general periods of high volatility are preceded by periods of high market drift, either upward or downward. After any appreciable market drift, the future market will be characterised by increased uncertainty while the price settles leading to an enhanced volatility. The correlations are more marked in non-trending markets, such as the GBP per USD exchange rate, than in say the USD per JPY rate. The correlations are strongest when longer periods, such as quarters, are used when the volatility estimate may be improved by as much as 20%. However the predictive power from shorter periods may be enhanced by suitable averaging.

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## References

1. M.T. Leung, H. Daouk, A. Chen, Int. J. Forcasting, **16**, 173 (2000).